

# **Pure Tunings**

author: Dominik Kruppa

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The aim of this article is to introduce three newly invented musical tunings. They were created in August 2012 by DK.

## Table of contents

Starting-point.....	3
Prime Tuning or Kruppa I Tuning.....	3
Pure Tuning Unbalanced or Kruppa II Tuning.....	3
Pure Tuning or Kruppa III Tuning.....	3
Mention about other possible tunings.....	4
The principle.....	4
Comparison with other temperaments.....	4
Werckmeister III.....	4
Lehman-Bach .....	5
Equal temperament.....	5
Conclusion.....	5
Appendix.....	7
Prime Tuning.....	8
Pure Tuning Unbalanced.....	13
Pure Tuning.....	18
Werckmeister III temperament.....	24
Lehman-Bach temperament.....	29
Equal temperament.....	34

## **Starting-point**

It is known that if you tune intervals as fractions of small whole numbers, the resulting interval will be pure, i.e. without beating. Tuning systems based on these intervals are called “just intonation”. However, these tunings were never widely used in classical music. The reasons for this are obvious – chords are very unbalanced and there are also some wolf intervals (so badly sounding intervals, so that they can't be used). These shortages make it effectively unusable.

I have invented the way how to overcome these flaws. If we use really clever numbers, we can build pure tuning system, that is balanced and without any unmusical wolf intervals. I will explain it on tunings in order I have invented them:

## **Prime Tuning or Kruppa I Tuning**

The idea is to use just prime numbers to build intervals, except for number 4. This will ensure that (almost) all intervals are unique, which will create vast number of tonal colours. It is striking at first glance. Ratios and cents used in this tuning as well as analysis of intervals are in Appendix.

I find this tuning very interesting and inspiring, although unbalanced. It sounds very good just in very specific music. Surely, new music can be written exactly for this tuning to make use of its strengths.

## **Pure Tuning Unbalanced or Kruppa II Tuning**

The idea is to replace some ratios from Prime Tuning for better balance and to make its worst intervals sound better. They are replaced by the multiplies of 2, 3, 5 and 19. Ratios and cents used in this tuning as well as analysis of intervals are in Appendix.

(The variant of this tuning is to tune tone Fis to  $45/32$ , which seems to be mathematically better, as it makes another two “ $3/2$ ” fifths, but musically it doesn't seem right; especially chord F# – A – C# is very dissonant, worse than it was.)

This tuning is something in between the other two – it has less tone colours than Prime Tuning, but more than Pure Tuning. It is more balanced than Prime Tuning, but less than Pure Tuning. It is usable with good result for more music than Prime Tuning, but not all, especially very complicated contrapunctual pieces and pieces using many black keys will sound strange.

## **Pure Tuning or Kruppa III Tuning**

The idea is to replace some ratios from Pure Tuning Unbalanced, so that too narrow fifths got wider and too wide major thirds get narrower. The replacement is done using multiplies of 2, 3, 5, 17 and 19, which creates sometimes a bit bigger numbers. Ratios and cents used in this tuning as well as analysis of intervals are in Appendix.

I find this tuning to be correct for all classical music, to be more exact, for music till the end of 19<sup>th</sup> century. There is also no problem with many modern pieces. It has tonal purity based on whole numbers and just intonation, while it is still playable in all keys and even suitable for the most complicated music.

## **Mention about other possible tunings**

It is certainly possible to go some steps further to produce, from mathematical point of view, even more balanced tuning. This would be accomplished by using even bigger whole numbers. But result is doubtful and I think musically incorrect. Can be something more balanced than balanced? Yes, it can be, at last, equal. Even equal temperament can be written as ratios of whole numbers, to be more exact, powers of 2 and 3, but that are enormously big numbers.

The bigger the number, the less pure the sound. And I wouldn't call tunings that use overly big numbers "pure".

## **The principle**

Up to these days, just  $5/4$  was considered as "pure major third". But I think that wording is misleading and incorrect. Pure Tuning (Kruppa III) uses as its "ideal" major third ratio  $24/19$ . This, being 404.44 cents wide, sounds pure. So there is not just one "pure major third", but they are more of them.

In temperaments known up to date, no one utilize another pure intervals than the few known in just intonation. That, of course, has produced insolvable problems. You just can't have all fifths of ratio  $5/4$  (386.31 cents) wide, it is impossible. And the more of them you have, the worse will be other intervals you create, especially fifths. Wolf intervals are often inevitable here.

The problem of creating tuning using just pure intervals is solved using richer variety of ratios that sound pure. This has also another pleasant consequence: There is considerably richer variety of tonal colours, or, it can be said, there are more sounds. This is in sharp contrast with equal temperament, which has all intervals in all keys the same.

I would say music that use more variety of intervals is more vivid.

## **Comparison with other temperaments**

I have chosen three reference temperaments for comparison. I haven't take into consideration any temperament or tuning which creates wolf intervals (such Just intonation or Meantone), because such temperament is not suitable for all keys, and therefore not for all music, not universally usable.

## **Werckmeister III**

I consider Werckmeister to be a model temperament, similar to many other which are called "Well temperaments". The strengths and shortages are similar among them.

Shortage of Werckmeister is that it contains some Pythagorean major thirds (407.82 cents). These intervals aren't unusable at all, but are so dissonant, so that it makes much music played in distant keys sound strange or even unplayable. It is right when composer has in mind where these thirds are located and compose accordingly. But when not, the result is often unpleasant.

It also has two contrasting sorts of fifth. One is  $3/2$ , second is considerably narrower, being 696.09 cents wide. That doesn't create ideal balance and it will sometimes in some music cause "unpleasant surprises".

At all, Werckmeister is very well usable, especially in organ music, but not universal and balanced enough to be suitable for all classical music.

Pure Tuning doesn't share its faults, as it has all major thirds considerably narrower (and purer), while fifths are more evenly distributed.

### **Lehman-Bach**

I think Lehman-Bach temperament is the one most similar to Pure Tuning, while still much different. If we compare intervals, we are not very wise of it, as they are very similar in its width. The most important intervals are major thirds and fifths. Major thirds are probably better and more balanced in Pure Tuning, as a result of dissonant E – G# third in Lehman-Bach, being 405.86 cents wide and doesn't sound very pleasant. Thirds of Pure tuning are considerably purer and more consonant. The fifths are undecided. Pure Tuning has two fifths a bit narrower than Lehman-Bach, but on the other hand, they are more evenly distributed and in sum closer to  $3/2$ , as a result of a fact, that A# – F in Lehman-Bach is wider than  $3/2$ , being 703.92 cents wide.

Lehman-Bach and Pure Tuning are both suitable for all classical music. But Pure Tuning, from its essence, produce purer intervals (all of them being ratios of integers), and in my ears it sounds really better musically. I would say it is more clear and brilliant.

### **Equal temperament**

Equal temperament is absolutely dominant today. Its popularity began in the beginning of the 20<sup>th</sup> century and was probably caused by the fact that all music is playable and there are no unpleasant “surprises” when playing in whatsoever key. All chord are the same, all keys are the same.

But there is a price to pay for it. Equal temperament sounds sterile. The concept of music played in different keys is almost lost, as all of them sounds completely the same, just a bit higher or lower in pitch. No one can now talk about different characteristics of each key, as it was in the past.

Moreover, equal temperament is mistuned all the round, there is a constant dissonance present in all chords that never resolves. Every major or minor third is, to some extent, dissonant. Perhaps most mistuned is minor seventh chord (also called dominant), which sounds much different from its pure ideal.

Equal temperament and Pure Tuning are in contrast with each other. Pure Tuning, as well as many historic temperaments, have more colours in itself, because it contains much more different intervals. It is also much more in tune, intervals are considerably purer. Someone not accustomed to historic temperaments will maybe get surprised from time to time, because some intervals will sound so different that he got used to; for example pure sixth contains vibrations not present in equal temperament, hence surprising. But this is just matter of getting used to.

As a summary, Equal temperament as well as Pure Tuning are suitable for all classical music, but Pure Tuning sounds more pure and vivid. On the other hand, Equal temperament sounds all the same in each situation.

### **Conclusion**

Pure Tuning proved to be good alternative to historic temperaments as well as to modern equal temperament. It can be, likewise, tuned on all classical instruments. Due to its universality and, comparing to equal temperament, purer tone quality it is recommended to replace it in a position of being the universal tuning for piano. In fact, it can satisfactorily replace equal temperament on each classical instrument or every

ensemble making it obsolete. Just when historically authentic performance is needed, some historic temperaments are more suitable, like Lehman-Bach for music of J. S. Bach and sons.

# Appendix

## Prime Tuning

C : 1 = 0  
C# :  $43/41 = 82.455300100817$   
D :  $19/17 = 192.5576066319$   
D# :  $13/11 = 289.20971940455$   
E :  $5/4 = 386.31371386483$   
F :  $4/3 = 498.04499913461$   
F# :  $7/5 = 582.51219260429$   
G :  $3/2 = 701.95500086539$   
G# :  $11/7 = 782.49203589563$   
A :  $5/3 = 884.35871299945$   
A# :  $23/13 = 987.7466854991$   
H :  $43/23 = 1083.2433583741$

statistics for interval minor second:

C -> C# :  $1 \rightarrow 43/41 = 43^*1 / 41^*1 = 43 / 41 = 82.455300100817$   
C# -> D :  $43/41 \rightarrow 19/17 = 19^*41 / 17^*43 = 779 / 731 = 110.10230653108$   
D -> D# :  $19/17 \rightarrow 13/11 = 13^*17 / 11^*19 = 221 / 209 = 96.652112772659$   
D# -> E :  $13/11 \rightarrow 5/4 = 5^*11 / 2^*2^*13 = 55 / 52 = 97.103994460281$   
E -> F :  $5/4 \rightarrow 4/3 = 2^*2^*2^*2 / 3^*5 = 16 / 15 = 111.73128526978$   
F -> F# :  $4/3 \rightarrow 7/5 = 7^*3 / 5^*2^*2 = 21 / 20 = 84.467193469678$   
F# -> G :  $7/5 \rightarrow 3/2 = 3^*5 / 2^*7 = 15 / 14 = 119.4428082611$   
G -> G# :  $3/2 \rightarrow 11/7 = 11^*2 / 7^*3 = 22 / 21 = 80.537035030244$   
G# -> A :  $11/7 \rightarrow 5/3 = 5^*7 / 3^*11 = 35 / 33 = 101.86667710382$   
A -> A# :  $5/3 \rightarrow 23/13 = 23^*3 / 13^*5 = 69 / 65 = 103.38797249966$   
A# -> H :  $23/13 \rightarrow 43/23 = 43^*13 / 23^*23 = 559 / 529 = 95.496672874997$   
H -> C :  $43/23 \rightarrow 1 = 2^*1^*23 / 1^*43 = 46 / 43 = 116.7566416259$

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1 x 80.5 1 x 82.5 1 x 84.5 1 x 95.5 1 x 96.7 1 x 97.1 1 x 101.9 1 x 103.4 1 x 110.1 1 x 111.7 1 x 116.8 1 x 119.4

statistics for interval major second:

C -> D :  $1 \rightarrow 19/17 = 19^*1 / 17^*1 = 19 / 17 = 192.5576066319$   
C# -> D# :  $43/41 \rightarrow 13/11 = 13^*41 / 11^*43 = 533 / 473 = 206.75441930374$   
D -> E :  $19/17 \rightarrow 5/4 = 5^*17 / 2^*2^*19 = 85 / 76 = 193.75610723294$   
D# -> F :  $13/11 \rightarrow 4/3 = 2^*2^*11 / 3^*13 = 44 / 39 = 208.83527973006$   
E -> F# :  $5/4 \rightarrow 7/5 = 7^*2^*2 / 5^*5 = 28 / 25 = 196.19847873946$   
F -> G :  $4/3 \rightarrow 3/2 = 3^*3 / 2^*2^*2 = 9 / 8 = 203.91000173077$   
F# -> G# :  $7/5 \rightarrow 11/7 = 11^*5 / 7^*7 = 55 / 49 = 199.97984329134$   
G -> A :  $3/2 \rightarrow 5/3 = 5^*2 / 3^*3 = 10 / 9 = 182.40371213406$   
G# -> A# :  $11/7 \rightarrow 23/13 = 23^*7 / 13^*11 = 161 / 143 = 205.25464960347$   
A -> H :  $5/3 \rightarrow 43/23 = 43^*3 / 23^*5 = 129 / 115 = 198.88464537465$   
A# -> C :  $23/13 \rightarrow 1 = 2^*1^*13 / 1^*23 = 26 / 23 = 212.2533145009$



$$H \rightarrow C\# : 43/23 \rightarrow 43/41 = 2^*43^*23 / 41^*43 = 46 / 41 = 199.21194172671$$

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1 x 182.4 1 x 192.6 1 x 193.8 1 x 196.2 1 x 198.9 1 x 199.2 1 x 200 1 x 203.9 1 x 205.3 1 x 206.8 1 x 208.8 1 x 212.3

statistics for interval minor third:

$$\begin{aligned} C \rightarrow D\# : 1 \rightarrow 13/11 &= 13^*1 / 11^*1 = 13 / 11 = 289.20971940455 \\ C\# \rightarrow E : 43/41 \rightarrow 5/4 &= 5^*41 / 2^*2^*43 = 205 / 172 = 303.85841376402 \\ D \rightarrow F : 19/17 \rightarrow 4/3 &= 2^*2^*17 / 3^*19 = 68 / 57 = 305.48739250272 \\ D\# \rightarrow F\# : 13/11 \rightarrow 7/5 &= 7^*11 / 5^*13 = 77 / 65 = 293.30247319974 \\ E \rightarrow G : 5/4 \rightarrow 3/2 &= 3^*2^*2 / 2^*5 = 6 / 5 = 315.64128700055 \\ F \rightarrow G\# : 4/3 \rightarrow 11/7 &= 11^*3 / 7^*2^*2 = 33 / 28 = 284.44703676102 \\ F\# \rightarrow A : 7/5 \rightarrow 5/3 &= 5^*5 / 3^*7 = 25 / 21 = 301.84652039516 \\ G \rightarrow A\# : 3/2 \rightarrow 23/13 &= 23^*2 / 13^*3 = 46 / 39 = 285.79168463372 \\ G\# \rightarrow H : 11/7 \rightarrow 43/23 &= 43^*7 / 23^*11 = 301 / 253 = 300.75132247847 \\ A \rightarrow C : 5/3 \rightarrow 1 &= 2^*1^*3 / 1^*5 = 6 / 5 = 315.64128700055 \\ A\# \rightarrow C\# : 23/13 \rightarrow 43/41 &= 2^*43^*13 / 41^*23 = 1118 / 943 = 294.70861460171 \\ H \rightarrow D : 43/23 \rightarrow 19/17 &= 2^*19^*23 / 17^*43 = 874 / 731 = 309.31424825779 \end{aligned}$$

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1 x 284.4 1 x 285.8 1 x 289.2 1 x 293.3 1 x 294.7 1 x 300.8 1 x 301.8 1 x 303.9 1 x 305.5 1 x 309.3 2 x 315.6

statistics for interval major third:

$$\begin{aligned} C \rightarrow E : 1 \rightarrow 5/4 &= 5^*1 / 2^*2^*1 = 5 / 4 = 386.31371386483 \\ C\# \rightarrow F : 43/41 \rightarrow 4/3 &= 2^*2^*41 / 3^*43 = 164 / 129 = 415.5896990338 \\ D \rightarrow F\# : 19/17 \rightarrow 7/5 &= 7^*17 / 5^*19 = 119 / 95 = 389.95458597239 \\ D\# \rightarrow G : 13/11 \rightarrow 3/2 &= 3^*11 / 2^*13 = 33 / 26 = 412.74528146083 \\ E \rightarrow G\# : 5/4 \rightarrow 11/7 &= 11^*2^*2 / 7^*5 = 44 / 35 = 396.1783220308 \\ F \rightarrow A : 4/3 \rightarrow 5/3 &= 5^*3 / 3^*2^*2 = 5 / 4 = 386.31371386483 \\ F\# \rightarrow A\# : 7/5 \rightarrow 23/13 &= 23^*5 / 13^*7 = 115 / 91 = 405.23449289481 \\ G \rightarrow H : 3/2 \rightarrow 43/23 &= 43^*2 / 23^*3 = 86 / 69 = 381.28835750871 \\ G\# \rightarrow C : 11/7 \rightarrow 1 &= 2^*1^*7 / 1^*11 = 14 / 11 = 417.50796410437 \\ A \rightarrow C\# : 5/3 \rightarrow 43/41 &= 2^*43^*3 / 41^*5 = 258 / 205 = 398.09658710137 \\ A\# \rightarrow D : 23/13 \rightarrow 19/17 &= 2^*19^*13 / 17^*23 = 494 / 391 = 404.81092113279 \\ H \rightarrow D\# : 43/23 \rightarrow 13/11 &= 2^*13^*23 / 11^*43 = 598 / 473 = 405.96636103045 \end{aligned}$$

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1 x 381.3 2 x 386.3 1 x 390 1 x 396.2 1 x 398.1 1 x 404.8 1 x 405.2 1 x 406 1 x 412.7 1 x 415.6 1 x 417.5

statistics for interval fourth:

$$\begin{aligned} C \rightarrow F : 1 \rightarrow 4/3 &= 2^*2^*1 / 3^*1 = 4 / 3 = 498.04499913461 \\ C\# \rightarrow F\# : 43/41 \rightarrow 7/5 &= 7^*41 / 5^*43 = 287 / 215 = 500.05689250347 \end{aligned}$$

D -> G :  $19/17 \rightarrow 3/2 = 3^{*17} / 2^{*19} = 51 / 38 = 509.39739423349$   
 D# -> G# :  $13/11 \rightarrow 11/7 = 11^{*11} / 7^{*13} = 121 / 91 = 493.28231649108$   
 E -> A :  $5/4 \rightarrow 5/3 = 5^{*2^2} / 3^{*5} = 4 / 3 = 498.04499913461$   
 F -> A# :  $4/3 \rightarrow 23/13 = 23^{*3} / 13^{*2^2} = 69 / 52 = 489.70168636449$   
 F# -> H :  $7/5 \rightarrow 43/23 = 43^{*5} / 23^{*7} = 215 / 161 = 500.73116576981$   
 G -> C :  $3/2 \rightarrow 1 = 2^{*1^2} / 1^{*3} = 4 / 3 = 498.04499913461$   
 G# -> C# :  $11/7 \rightarrow 43/41 = 2^{*43^7} / 41^{*11} = 602 / 451 = 499.96326420519$   
 A -> D :  $5/3 \rightarrow 19/17 = 2^{*19^3} / 17^{*5} = 114 / 85 = 508.19889363245$   
 A# -> D# :  $23/13 \rightarrow 13/11 = 2^{*13^13} / 11^{*23} = 338 / 253 = 501.46303390545$   
 H -> E :  $43/23 \rightarrow 5/4 = 2^{*5^23} / 2^{*2^43} = 230 / 172 = 503.07035549073$

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1 x 489.7 1 x 493.3 3 x 498 1 x 500 1 x 500.1 1 x 500.7 1 x 501.5 1 x 503.1 1 x  
 508.2 1 x 509.4

statistics for interval tritone:

C -> F# :  $1 \rightarrow 7/5 = 7^{*1} / 5^{*1} = 7 / 5 = 582.51219260429$   
 C# -> G :  $43/41 \rightarrow 3/2 = 3^{*41} / 2^{*43} = 123 / 86 = 619.49970076457$   
 D -> G# :  $19/17 \rightarrow 11/7 = 11^{*17} / 7^{*19} = 187 / 133 = 589.93442926374$   
 D# -> A :  $13/11 \rightarrow 5/3 = 5^{*11} / 3^{*13} = 55 / 39 = 595.14899359489$   
 E -> A# :  $5/4 \rightarrow 23/13 = 23^{*2^2} / 13^{*5} = 92 / 65 = 601.43297163427$   
 F -> H :  $4/3 \rightarrow 43/23 = 43^{*3} / 23^{*2^2} = 129 / 92 = 585.19835923949$   
 F# -> C :  $7/5 \rightarrow 1 = 2^{*1^5} / 1^{*7} = 10 / 7 = 617.48780739571$   
 G -> C# :  $3/2 \rightarrow 43/41 = 2^{*43^2} / 41^{*3} = 172 / 123 = 580.50029923543$   
 G# -> D :  $11/7 \rightarrow 19/17 = 2^{*19^7} / 17^{*11} = 266 / 187 = 610.06557073626$   
 A -> D# :  $5/3 \rightarrow 13/11 = 2^{*13^3} / 11^{*5} = 78 / 55 = 604.85100640511$   
 A# -> E :  $23/13 \rightarrow 5/4 = 2^{*5^13} / 2^{*2^23} = 130 / 92 = 598.56702836573$   
 H -> F :  $43/23 \rightarrow 4/3 = 2^{*2^2^23} / 3^{*43} = 184 / 129 = 614.80164076051$

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1 x 580.5 1 x 582.5 1 x 585.2 1 x 589.9 1 x 595.1 1 x 598.6 1 x 601.4 1 x 604.9 1 x  
 610.1 1 x 614.8 1 x 617.5 1 x 619.5

statistics for interval fifth:

C -> G :  $1 \rightarrow 3/2 = 3^{*1} / 2^{*1} = 3 / 2 = 701.95500086539$   
 C# -> G# :  $43/41 \rightarrow 11/7 = 11^{*41} / 7^{*43} = 451 / 301 = 700.03673579481$   
 D -> A :  $19/17 \rightarrow 5/3 = 5^{*17} / 3^{*19} = 85 / 57 = 691.80110636755$   
 D# -> A# :  $13/11 \rightarrow 23/13 = 23^{*11} / 13^{*13} = 253 / 169 = 698.53696609455$   
 E -> H :  $5/4 \rightarrow 43/23 = 43^{*2^2} / 23^{*5} = 172 / 115 = 696.92964450927$   
 F -> C :  $4/3 \rightarrow 1 = 2^{*1^3} / 1^{*2^2} = 6 / 4 = 701.95500086539$   
 F# -> C# :  $7/5 \rightarrow 43/41 = 2^{*43^5} / 41^{*7} = 430 / 287 = 699.94310749653$   
 G -> D :  $3/2 \rightarrow 19/17 = 2^{*19^2} / 17^{*3} = 76 / 51 = 690.60260576651$   
 G# -> D# :  $11/7 \rightarrow 13/11 = 2^{*13^7} / 11^{*11} = 182 / 121 = 706.71768350892$   
 A -> E :  $5/3 \rightarrow 5/4 = 2^{*5^3} / 2^{*2^5} = 6 / 4 = 701.95500086539$   
 A# -> F :  $23/13 \rightarrow 4/3 = 2^{*2^2^13} / 3^{*23} = 104 / 69 = 710.29831363551$   
 H -> F# :  $43/23 \rightarrow 7/5 = 2^{*7^23} / 5^{*43} = 322 / 215 = 699.26883423019$

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1 x 690.6 1 x 691.8 1 x 696.9 1 x 698.5 1 x 699.3 1 x 699.9 1 x 700 3 x 702 1 x  
706.7 1 x 710.3

statistics for interval minor sixth:

C -> G# : 1 ->  $11/7 = 11^*1 / 7^*1 = 11 / 7 = 782.49203589563$   
C# -> A :  $43/41 -> 5/3 = 5^*41 / 3^*43 = 205 / 129 = 801.90341289863$   
D -> A# :  $19/17 -> 23/13 = 23^*17 / 13^*19 = 391 / 247 = 795.18907886721$   
D# -> H :  $13/11 -> 43/23 = 43^*11 / 23^*13 = 473 / 299 = 794.03363896955$   
E -> C :  $5/4 -> 1 = 2^*1^*2^*2 / 1^*5 = 8 / 5 = 813.68628613517$   
F -> C# :  $4/3 -> 43/41 = 2^*43^*3 / 41^*2^*2 = 258 / 164 = 784.4103009662$   
F# -> D :  $7/5 -> 19/17 = 2^*19^*5 / 17^*7 = 190 / 119 = 810.04541402761$   
G -> D# :  $3/2 -> 13/11 = 2^*13^*2 / 11^*3 = 52 / 33 = 787.25471853917$   
G# -> E :  $11/7 -> 5/4 = 2^*5^*7 / 2^*2^*11 = 70 / 44 = 803.8216779692$   
A -> F :  $5/3 -> 4/3 = 2^*2^*2^*3 / 3^*5 = 8 / 5 = 813.68628613517$   
A# -> F# :  $23/13 -> 7/5 = 2^*7^*13 / 5^*23 = 182 / 115 = 794.76550710519$   
H -> G :  $43/23 -> 3/2 = 2^*3^*23 / 2^*43 = 138 / 86 = 818.71164249129$

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1 x 782.5 1 x 784.4 1 x 787.3 1 x 794 1 x 794.8 1 x 795.2 1 x 801.9 1 x 803.8 1 x  
810 2 x 813.7 1 x 818.7

statistics for interval major sixth:

C -> A : 1 ->  $5/3 = 5^*1 / 3^*1 = 5 / 3 = 884.35871299945$   
C# -> A# :  $43/41 -> 23/13 = 23^*41 / 13^*43 = 943 / 559 = 905.29138539829$   
D -> H :  $19/17 -> 43/23 = 43^*17 / 23^*19 = 731 / 437 = 890.68575174221$   
D# -> C :  $13/11 -> 1 = 2^*1^*11 / 1^*13 = 22 / 13 = 910.79028059545$   
E -> C# :  $5/4 -> 43/41 = 2^*43^*2^*2 / 41^*5 = 344 / 205 = 896.14158623598$   
F -> D :  $4/3 -> 19/17 = 2^*19^*3 / 17^*2^*2 = 114 / 68 = 894.51260749728$   
F# -> D# :  $7/5 -> 13/11 = 2^*13^*5 / 11^*7 = 130 / 77 = 906.69752680026$   
G -> E :  $3/2 -> 5/4 = 2^*5^*2 / 2^*2^*3 = 10 / 6 = 884.35871299945$   
G# -> F :  $11/7 -> 4/3 = 2^*2^*2^*7 / 3^*11 = 56 / 33 = 915.55296323898$   
A -> F# :  $5/3 -> 7/5 = 2^*7^*3 / 5^*5 = 42 / 25 = 898.15347960484$   
A# -> G :  $23/13 -> 3/2 = 2^*3^*13 / 2^*23 = 78 / 46 = 914.20831536628$   
H -> G# :  $43/23 -> 11/7 = 2^*11^*23 / 7^*43 = 506 / 301 = 899.24867752153$

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2 x 884.4 1 x 890.7 1 x 894.5 1 x 896.1 1 x 898.2 1 x 899.2 1 x 905.3 1 x 906.7 1 x  
910.8 1 x 914.2 1 x 915.6

statistics for interval minor seventh:

C -> A# : 1 ->  $23/13 = 23^*1 / 13^*1 = 23 / 13 = 987.7466854991$   
C# -> H :  $43/41 -> 43/23 = 43^*41 / 23^*43 = 41 / 23 = 1000.7880582733$   
D -> C :  $19/17 -> 1 = 2^*1^*17 / 1^*19 = 34 / 19 = 1007.4423933681$   
D# -> C# :  $13/11 -> 43/41 = 2^*43^*11 / 41^*13 = 946 / 533 = 993.24558069626$   
E -> D :  $5/4 -> 19/17 = 2^*19^*2^*2 / 17^*5 = 152 / 85 = 1006.2438927671$

F -> D# :  $4/3 \rightarrow 13/11 = 2^*13^*3 / 11^*2^*2 = 78 / 44 = 991.16472026994$   
 F# -> E :  $7/5 \rightarrow 5/4 = 2^*5^*5 / 2^*2^*7 = 50 / 28 = 1003.8015212605$   
 G -> F :  $3/2 \rightarrow 4/3 = 2^*2^*2^*2 / 3^*3 = 16 / 9 = 996.08999826923$   
 G# -> F# :  $11/7 \rightarrow 7/5 = 2^*7^*7 / 5^*11 = 98 / 55 = 1000.0201567087$   
 A -> G :  $5/3 \rightarrow 3/2 = 2^*3^*3 / 2^*5 = 18 / 10 = 1017.5962878659$   
 A# -> G# :  $23/13 \rightarrow 11/7 = 2^*11^*13 / 7^*23 = 286 / 161 = 994.74535039653$   
 H -> A :  $43/23 \rightarrow 5/3 = 2^*5^*23 / 3^*43 = 230 / 129 = 1001.1153546253$

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1 x 987.7 1 x 991.2 1 x 993.2 1 x 994.7 1 x 996.1 1 x 1000 1 x 1000.8 1 x 1001.1 1 x  
 1003.8 1 x 1006.2 1 x 1007.4 1 x 1017.6

statistics for interval major seventh:

C -> H :  $1 \rightarrow 43/23 = 43^*1 / 23^*1 = 43 / 23 = 1083.2433583741$   
 C# -> C :  $43/41 \rightarrow 1 = 2^*1^*41 / 1^*43 = 82 / 43 = 1117.5446998992$   
 D -> C# :  $19/17 \rightarrow 43/41 = 2^*43^*17 / 41^*19 = 1462 / 779 = 1089.8976934689$   
 D# -> D :  $13/11 \rightarrow 19/17 = 2^*19^*11 / 17^*13 = 418 / 221 = 1103.3478872273$   
 E -> D# :  $5/4 \rightarrow 13/11 = 2^*13^*2^*2 / 11^*5 = 104 / 55 = 1102.8960055397$   
 F -> E :  $4/3 \rightarrow 5/4 = 2^*5^*3 / 2^*2^*2^*2 = 30 / 16 = 1088.2687147302$   
 F# -> F :  $7/5 \rightarrow 4/3 = 2^*2^*2^*5 / 3^*7 = 40 / 21 = 1115.5328065303$   
 G -> F# :  $3/2 \rightarrow 7/5 = 2^*7^*2 / 5^*3 = 28 / 15 = 1080.5571917389$   
 G# -> G :  $11/7 \rightarrow 3/2 = 2^*3^*7 / 2^*11 = 42 / 22 = 1119.4629649698$   
 A -> G# :  $5/3 \rightarrow 11/7 = 2^*11^*3 / 7^*5 = 66 / 35 = 1098.1333228962$   
 A# -> A :  $23/13 \rightarrow 5/3 = 2^*5^*13 / 3^*23 = 130 / 69 = 1096.6120275003$   
 H -> A# :  $43/23 \rightarrow 23/13 = 2^*23^*23 / 13^*43 = 1058 / 559 = 1104.503327125$

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1 x 1080.6 1 x 1083.2 1 x 1088.3 1 x 1089.9 1 x 1096.6 1 x 1098.1 1 x 1102.9 1 x  
 1103.3 1 x 1104.5 1 x 1115.5 1 x 1117.5 1 x 1119.5

## Pure Tuning Unbalanced

C : 1 = 0  
C# : 20/19 = 88.800697732532  
D : 19/17 = 192.5576066319  
D# : 45/38 = 292.71069946331  
E : 5/4 = 386.31371386483  
F : 4/3 = 498.04499913461  
F# : 24/17 = 596.99959136498  
G : 3/2 = 701.95500086539  
G# : 30/19 = 790.75569859792  
A : 5/3 = 884.35871299945  
A# : 16/9 = 996.08999826923  
H : 15/8 = 1088.2687147302

statistics for interval minor second:

C -> C# : 1 -> 20/19 =  $2^2 \cdot 5^1 / 19^1 = 20 / 19 = 88.800697732532$   
C# -> D : 20/19 -> 19/17 =  $19^1 \cdot 19 / 17^2 \cdot 2^2 \cdot 5 = 361 / 340 = 103.75690889936$   
D -> D# : 19/17 -> 45/38 =  $3^3 \cdot 5^1 \cdot 17 / 2^1 \cdot 19^1 \cdot 19 = 765 / 722 = 100.15309283141$   
D# -> E : 45/38 -> 5/4 =  $5^2 \cdot 2^1 \cdot 19 / 2^2 \cdot 3^3 \cdot 5 = 19 / 18 = 93.603014401528$   
E -> F : 5/4 -> 4/3 =  $2^2 \cdot 2^2 \cdot 2 / 3^3 \cdot 5 = 16 / 15 = 111.73128526978$   
F -> F# : 4/3 -> 24/17 =  $2^2 \cdot 2^2 \cdot 3^3 \cdot 3 / 17^2 \cdot 2^2 = 18 / 17 = 98.954592230368$   
F# -> G : 24/17 -> 3/2 =  $3^3 \cdot 17 / 2^2 \cdot 2^2 \cdot 2^2 \cdot 3 = 17 / 16 = 104.95540950041$   
G -> G# : 3/2 -> 30/19 =  $2^3 \cdot 3^5 \cdot 2 / 19^3 = 20 / 19 = 88.800697732532$   
G# -> A : 30/19 -> 5/3 =  $5^1 \cdot 19 / 3^2 \cdot 3^3 \cdot 5 = 19 / 18 = 93.603014401528$   
A -> A# : 5/3 -> 16/9 =  $2^2 \cdot 2^2 \cdot 2^2 \cdot 3 / 3^3 \cdot 3^3 \cdot 5 = 16 / 15 = 111.73128526978$   
A# -> H : 16/9 -> 15/8 =  $3^5 \cdot 3^3 \cdot 3 / 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 \cdot 2 = 135 / 128 = 92.178716460997$   
H -> C : 15/8 -> 1 =  $2^1 \cdot 1^2 \cdot 2^2 \cdot 2 / 1^3 \cdot 3^5 = 16 / 15 = 111.73128526978$

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2 x 88.8 1 x 92.2 2 x 93.6 1 x 99 1 x 100.2 1 x 103.8 1 x 105 3 x 111.7

statistics for interval major second:

C -> D : 1 -> 19/17 =  $19^1 / 17^1 = 19 / 17 = 192.5576066319$   
C# -> D# : 20/19 -> 45/38 =  $3^3 \cdot 3^5 \cdot 19 / 2^1 \cdot 19^2 \cdot 2^2 \cdot 5 = 9 / 8 = 203.91000173077$   
D -> E : 19/17 -> 5/4 =  $5^1 \cdot 17 / 2^2 \cdot 2^1 \cdot 19 = 85 / 76 = 193.75610723294$   
D# -> F : 45/38 -> 4/3 =  $2^2 \cdot 2^2 \cdot 19 / 3^3 \cdot 3^3 \cdot 5 = 152 / 135 = 205.33429967131$   
E -> F# : 5/4 -> 24/17 =  $2^2 \cdot 2^2 \cdot 3^3 \cdot 2^2 / 17^2 \cdot 5 = 96 / 85 = 210.68587750015$   
F -> G : 4/3 -> 3/2 =  $3^3 / 2^2 \cdot 2^2 = 9 / 8 = 203.91000173077$   
F# -> G# : 24/17 -> 30/19 =  $2^3 \cdot 3^5 \cdot 17 / 19^2 \cdot 2^2 \cdot 2^2 \cdot 3 = 85 / 76 = 193.75610723294$   
G -> A : 3/2 -> 5/3 =  $5^2 / 3^3 = 10 / 9 = 182.40371213406$   
G# -> A# : 30/19 -> 16/9 =  $2^2 \cdot 2^2 \cdot 2^1 \cdot 19 / 3^3 \cdot 3^2 \cdot 3^3 \cdot 5 = 152 / 135 = 205.33429967131$   
A -> H : 5/3 -> 15/8 =  $3^5 \cdot 5^3 / 2^2 \cdot 2^2 \cdot 5 = 9 / 8 = 203.91000173077$   
A# -> C : 16/9 -> 1 =  $2^1 \cdot 1^3 \cdot 3^3 / 1^2 \cdot 2^2 \cdot 2^2 \cdot 2 = 18 / 16 = 203.91000173077$   
H -> C# : 15/8 -> 20/19 =  $2^2 \cdot 2^2 \cdot 5^2 \cdot 2^2 \cdot 2 / 19^3 \cdot 3^5 = 64 / 57 = 200.53198300231$

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1 x 182.4 1 x 192.6 2 x 193.8 1 x 200.5 4 x 203.9 2 x 205.3 1 x 210.7

statistics for interval minor third:

C -> D# :  $1 \rightarrow 45/38 = 3^3 \cdot 5^1 / 2^1 \cdot 19^1 = 45 / 38 = 292.71069946331$   
C# -> E :  $20/19 \rightarrow 5/4 = 5^1 \cdot 19 / 2^2 \cdot 2^2 \cdot 5 = 19 / 16 = 297.5130161323$   
D -> F :  $19/17 \rightarrow 4/3 = 2^2 \cdot 17 / 3^1 \cdot 19 = 68 / 57 = 305.48739250272$   
D# -> F# :  $45/38 \rightarrow 24/17 = 2^2 \cdot 2^2 \cdot 3^1 \cdot 2^1 \cdot 19 / 17^1 \cdot 3^1 \cdot 3^1 \cdot 5 = 304 / 255 = 304.28889190167$   
E -> G :  $5/4 \rightarrow 3/2 = 3^1 \cdot 2^2 / 2^1 \cdot 5 = 6 / 5 = 315.64128700055$   
F -> G# :  $4/3 \rightarrow 30/19 = 2^2 \cdot 3^1 \cdot 5^1 \cdot 3 / 19^1 \cdot 2^2 = 45 / 38 = 292.71069946331$   
F# -> A :  $24/17 \rightarrow 5/3 = 5^1 \cdot 17 / 3^1 \cdot 2^2 \cdot 2^2 \cdot 3 = 85 / 72 = 287.35912163447$   
G -> A# :  $3/2 \rightarrow 16/9 = 2^2 \cdot 2^2 \cdot 2^2 / 3^1 \cdot 3^1 \cdot 3 = 32 / 27 = 294.13499740384$   
G# -> H :  $30/19 \rightarrow 15/8 = 3^1 \cdot 5^1 \cdot 19 / 2^2 \cdot 2^2 \cdot 2^1 \cdot 3^1 \cdot 5 = 19 / 16 = 297.5130161323$   
A -> C :  $5/3 \rightarrow 1 = 2^1 \cdot 1^1 \cdot 3 / 1^1 \cdot 5 = 6 / 5 = 315.64128700055$   
A# -> C# :  $16/9 \rightarrow 20/19 = 2^2 \cdot 2^2 \cdot 2^1 \cdot 5^1 \cdot 3^1 \cdot 3 / 19^1 \cdot 2^2 \cdot 2^2 \cdot 2 = 90 / 76 = 292.71069946331$   
H -> D :  $15/8 \rightarrow 19/17 = 2^1 \cdot 19^1 \cdot 2^2 \cdot 2^2 / 17^1 \cdot 3^1 \cdot 5 = 304 / 255 = 304.28889190167$

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1 x 287.4 3 x 292.7 1 x 294.1 2 x 297.5 2 x 304.3 1 x 305.5 2 x 315.6

statistics for interval major third:

C -> E :  $1 \rightarrow 5/4 = 5^1 / 2^2 \cdot 2^1 = 5 / 4 = 386.31371386483$   
C# -> F :  $20/19 \rightarrow 4/3 = 2^2 \cdot 2^1 \cdot 19 / 3^1 \cdot 2^2 \cdot 2^1 \cdot 5 = 19 / 15 = 409.24430140208$   
D -> F# :  $19/17 \rightarrow 24/17 = 2^2 \cdot 2^2 \cdot 2^1 \cdot 3^1 \cdot 17 / 17^1 \cdot 19 = 24 / 19 = 404.44198473308$   
D# -> G :  $45/38 \rightarrow 3/2 = 3^1 \cdot 2^1 \cdot 19 / 2^1 \cdot 3^1 \cdot 3^1 \cdot 5 = 19 / 15 = 409.24430140208$   
E -> G# :  $5/4 \rightarrow 30/19 = 2^2 \cdot 3^1 \cdot 5^1 \cdot 2^2 / 19^1 \cdot 5 = 24 / 19 = 404.44198473308$   
F -> A :  $4/3 \rightarrow 5/3 = 5^1 \cdot 3 / 3^1 \cdot 2^2 = 5 / 4 = 386.31371386483$   
F# -> A# :  $24/17 \rightarrow 16/9 = 2^2 \cdot 2^2 \cdot 2^1 \cdot 17 / 3^1 \cdot 3^1 \cdot 2^2 \cdot 2^1 \cdot 3 = 34 / 27 = 399.09040690424$   
G -> H :  $3/2 \rightarrow 15/8 = 3^1 \cdot 5^1 \cdot 2 / 2^2 \cdot 2^2 \cdot 2^1 \cdot 3 = 5 / 4 = 386.31371386483$   
G# -> C :  $30/19 \rightarrow 1 = 2^1 \cdot 1^1 \cdot 19 / 1^1 \cdot 2^1 \cdot 3^1 \cdot 5 = 38 / 30 = 409.24430140208$   
A -> C# :  $5/3 \rightarrow 20/19 = 2^2 \cdot 2^2 \cdot 2^1 \cdot 5^1 \cdot 3 / 19^1 \cdot 5 = 24 / 19 = 404.44198473308$   
A# -> D :  $16/9 \rightarrow 19/17 = 2^1 \cdot 19^1 \cdot 3^1 \cdot 3 / 17^1 \cdot 2^2 \cdot 2^2 \cdot 2 = 342 / 272 = 396.46760836267$   
H -> D# :  $15/8 \rightarrow 45/38 = 2^1 \cdot 3^1 \cdot 3^1 \cdot 5^1 \cdot 2^2 \cdot 2^2 / 2^1 \cdot 19^1 \cdot 3^1 \cdot 5 = 24 / 19 = 404.44198473308$

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3 x 386.3 1 x 396.5 1 x 399.1 4 x 404.4 3 x 409.2

statistics for interval fourth:

C -> F :  $1 \rightarrow 4/3 = 2^2 \cdot 2^1 / 3^1 \cdot 1 = 4 / 3 = 498.04499913461$   
C# -> F# :  $20/19 \rightarrow 24/17 = 2^2 \cdot 2^2 \cdot 3^1 \cdot 19 / 17^1 \cdot 2^2 \cdot 2^1 \cdot 5 = 114 / 85 = 508.19889363245$   
D -> G :  $19/17 \rightarrow 3/2 = 3^1 \cdot 17 / 2^1 \cdot 19 = 51 / 38 = 509.39739423349$   
D# -> G# :  $45/38 \rightarrow 30/19 = 2^2 \cdot 3^1 \cdot 5^1 \cdot 2^1 \cdot 19 / 19^1 \cdot 3^1 \cdot 3^1 \cdot 5 = 4 / 3 = 498.04499913461$   
E -> A :  $5/4 \rightarrow 5/3 = 5^1 \cdot 2^2 / 3^1 \cdot 5 = 4 / 3 = 498.04499913461$   
F -> A# :  $4/3 \rightarrow 16/9 = 2^2 \cdot 2^2 \cdot 2^1 \cdot 3 / 3^1 \cdot 3^1 \cdot 2^2 = 4 / 3 = 498.04499913461$   
F# -> H :  $24/17 \rightarrow 15/8 = 3^1 \cdot 5^1 \cdot 17 / 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^1 \cdot 3 = 85 / 64 = 491.26912336524$

$G \rightarrow C: 3/2 \rightarrow 1 = 2^1 \cdot 2 / 1^1 \cdot 3 = 4 / 3 = 498.04499913461$   
 $G\# \rightarrow C\#: 30/19 \rightarrow 20/19 = 2^2 \cdot 2^2 \cdot 5^1 \cdot 19 / 19^1 \cdot 2^1 \cdot 3^1 \cdot 5 = 4 / 3 = 498.04499913461$   
 $A \rightarrow D: 5/3 \rightarrow 19/17 = 2^1 \cdot 19^1 \cdot 3 / 17^1 \cdot 5 = 114 / 85 = 508.19889363245$   
 $A\# \rightarrow D\#: 16/9 \rightarrow 45/38 = 2^3 \cdot 3^3 \cdot 5^1 \cdot 3^1 \cdot 3 / 2^1 \cdot 19^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 = 810 / 608 = 496.62070119408$   
 $H \rightarrow E: 15/8 \rightarrow 5/4 = 2^5 \cdot 2^1 \cdot 2^1 \cdot 2^1 / 2^1 \cdot 2^1 \cdot 3^1 \cdot 5 = 4 / 3 = 498.04499913461$

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1 x 491.3 1 x 496.6 7 x 498 2 x 508.2 1 x 509.4

statistics for interval tritone:

$C \rightarrow F\#: 1 \rightarrow 24/17 = 2^2 \cdot 2^1 \cdot 3^1 \cdot 1 / 17^1 \cdot 1 = 24 / 17 = 596.99959136498$   
 $C\# \rightarrow G: 20/19 \rightarrow 3/2 = 3^1 \cdot 19 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 5 = 57 / 40 = 613.15430313286$   
 $D \rightarrow G\#: 19/17 \rightarrow 30/19 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 17 / 19^1 \cdot 19 = 510 / 361 = 598.19809196602$   
 $D\# \rightarrow A: 45/38 \rightarrow 5/3 = 5^1 \cdot 2^1 \cdot 19 / 3^1 \cdot 3^1 \cdot 3^1 \cdot 5 = 38 / 27 = 591.64801353614$   
 $E \rightarrow A\#: 5/4 \rightarrow 16/9 = 2^2 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 / 3^1 \cdot 3^1 \cdot 5 = 64 / 45 = 609.77628440439$   
 $F \rightarrow H: 4/3 \rightarrow 15/8 = 3^1 \cdot 5^1 \cdot 3 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2 = 45 / 32 = 590.22371559561$   
 $F\# \rightarrow C: 24/17 \rightarrow 1 = 2^1 \cdot 1^1 \cdot 17 / 1^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3 = 34 / 24 = 603.00040863502$   
 $G \rightarrow C\#: 3/2 \rightarrow 20/19 = 2^2 \cdot 2^1 \cdot 2^1 \cdot 5^1 \cdot 2 / 19^1 \cdot 3 = 80 / 57 = 586.84569686714$   
 $G\# \rightarrow D: 30/19 \rightarrow 19/17 = 2^1 \cdot 19^1 \cdot 19 / 17^1 \cdot 2^1 \cdot 3^1 \cdot 5 = 722 / 510 = 601.80190803398$   
 $A \rightarrow D\#: 5/3 \rightarrow 45/38 = 2^1 \cdot 3^1 \cdot 3^1 \cdot 5^1 \cdot 3 / 2^1 \cdot 19^1 \cdot 5 = 54 / 38 = 608.35198646386$   
 $A\# \rightarrow E: 16/9 \rightarrow 5/4 = 2^1 \cdot 5^1 \cdot 3^1 \cdot 3 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 = 90 / 64 = 590.22371559561$   
 $H \rightarrow F: 15/8 \rightarrow 4/3 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2 / 3^1 \cdot 3^1 \cdot 5 = 64 / 45 = 609.77628440439$

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1 x 586.8 2 x 590.2 1 x 591.6 1 x 597 1 x 598.2 1 x 601.8 1 x 603 1 x 608.4 2 x 609.8 1 x 613.2

statistics for interval fifth:

$C \rightarrow G: 1 \rightarrow 3/2 = 3^1 \cdot 1 / 2^1 \cdot 1 = 3 / 2 = 701.95500086539$   
 $C\# \rightarrow G\#: 20/19 \rightarrow 30/19 = 2^1 \cdot 3^1 \cdot 5^1 \cdot 19 / 19^1 \cdot 2^1 \cdot 2^1 \cdot 5 = 3 / 2 = 701.95500086539$   
 $D \rightarrow A: 19/17 \rightarrow 5/3 = 5^1 \cdot 17 / 3^1 \cdot 19 = 85 / 57 = 691.80110636755$   
 $D\# \rightarrow A\#: 45/38 \rightarrow 16/9 = 2^2 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 19 / 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 5 = 608 / 405 = 703.37929880592$   
 $E \rightarrow H: 5/4 \rightarrow 15/8 = 3^1 \cdot 5^1 \cdot 2^1 \cdot 2 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 5 = 3 / 2 = 701.95500086539$   
 $F \rightarrow C: 4/3 \rightarrow 1 = 2^1 \cdot 1^1 \cdot 3 / 1^1 \cdot 2^1 \cdot 2 = 6 / 4 = 701.95500086539$   
 $F\# \rightarrow C\#: 24/17 \rightarrow 20/19 = 2^2 \cdot 2^1 \cdot 2^1 \cdot 5^1 \cdot 17 / 19^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3 = 170 / 114 = 691.80110636755$   
 $G \rightarrow D: 3/2 \rightarrow 19/17 = 2^1 \cdot 19^1 \cdot 2 / 17^1 \cdot 3 = 76 / 51 = 690.60260576651$   
 $G\# \rightarrow D\#: 30/19 \rightarrow 45/38 = 2^1 \cdot 3^1 \cdot 3^1 \cdot 5^1 \cdot 19 / 2^1 \cdot 19^1 \cdot 2^1 \cdot 3^1 \cdot 5 = 6 / 4 = 701.95500086539$   
 $A \rightarrow E: 5/3 \rightarrow 5/4 = 2^1 \cdot 5^1 \cdot 3 / 2^1 \cdot 2^1 \cdot 5 = 6 / 4 = 701.95500086539$   
 $A\# \rightarrow F: 16/9 \rightarrow 4/3 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 3 / 3^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2 = 6 / 4 = 701.95500086539$   
 $H \rightarrow F\#: 15/8 \rightarrow 24/17 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 / 17^1 \cdot 3^1 \cdot 5 = 128 / 85 = 708.73087663476$

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1 x 690.6 2 x 691.8 7 x 702 1 x 703.4 1 x 708.7

statistics for interval minor sixth:

C -> G# :  $1 \rightarrow 30/19 = 2^*3^*5^*1 / 19^*1 = 30 / 19 = 790.75569859792$   
C# -> A :  $20/19 \rightarrow 5/3 = 5^*19 / 3^*2^*2^*5 = 19 / 12 = 795.55801526692$   
D -> A# :  $19/17 \rightarrow 16/9 = 2^*2^*2^*2^*17 / 3^*3^*19 = 272 / 171 = 803.53239163733$   
D# -> H :  $45/38 \rightarrow 15/8 = 3^*5^*2^*19 / 2^*2^*2^*3^*3^*5 = 19 / 12 = 795.55801526692$   
E -> C :  $5/4 \rightarrow 1 = 2^*1^*2^*2 / 1^*5 = 8 / 5 = 813.68628613517$   
F -> C# :  $4/3 \rightarrow 20/19 = 2^*2^*2^*5^*3 / 19^*2^*2 = 30 / 19 = 790.75569859792$   
F# -> D :  $24/17 \rightarrow 19/17 = 2^*19^*17 / 17^*2^*2^*2^*3 = 38 / 24 = 795.55801526692$   
G -> D# :  $3/2 \rightarrow 45/38 = 2^*3^*3^*5^*2 / 2^*19^*3 = 30 / 19 = 790.75569859792$   
G# -> E :  $30/19 \rightarrow 5/4 = 2^*5^*19 / 2^*2^*2^*3^*5 = 38 / 24 = 795.55801526692$   
A -> F :  $5/3 \rightarrow 4/3 = 2^*2^*2^*3 / 3^*5 = 8 / 5 = 813.68628613517$   
A# -> F# :  $16/9 \rightarrow 24/17 = 2^*2^*2^*2^*3^*3^*3 / 17^*2^*2^*2^*2 = 54 / 34 = 800.90959309576$   
H -> G :  $15/8 \rightarrow 3/2 = 2^*3^*2^*2^*2 / 2^*3^*5 = 8 / 5 = 813.68628613517$

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3 x 790.8 4 x 795.6 1 x 800.9 1 x 803.5 3 x 813.7

statistics for interval major sixth:

C -> A :  $1 \rightarrow 5/3 = 5^*1 / 3^*1 = 5 / 3 = 884.35871299945$   
C# -> A# :  $20/19 \rightarrow 16/9 = 2^*2^*2^*2^*19 / 3^*3^*2^*2^*5 = 76 / 45 = 907.28930053669$   
D -> H :  $19/17 \rightarrow 15/8 = 3^*5^*17 / 2^*2^*2^*19 = 255 / 152 = 895.71110809833$   
D# -> C :  $45/38 \rightarrow 1 = 2^*1^*2^*19 / 1^*3^*3^*5 = 76 / 45 = 907.28930053669$   
E -> C# :  $5/4 \rightarrow 20/19 = 2^*2^*2^*5^*2^*2 / 19^*5 = 32 / 19 = 902.4869838677$   
F -> D :  $4/3 \rightarrow 19/17 = 2^*19^*3 / 17^*2^*2 = 114 / 68 = 894.51260749728$   
F# -> D# :  $24/17 \rightarrow 45/38 = 2^*3^*3^*5^*17 / 2^*19^*2^*2^*2^*3 = 510 / 304 = 895.71110809833$   
G -> E :  $3/2 \rightarrow 5/4 = 2^*5^*2 / 2^*2^*3 = 10 / 6 = 884.35871299945$   
G# -> F :  $30/19 \rightarrow 4/3 = 2^*2^*2^*19 / 3^*2^*3^*5 = 76 / 45 = 907.28930053669$   
A -> F# :  $5/3 \rightarrow 24/17 = 2^*2^*2^*2^*3^*3 / 17^*5 = 144 / 85 = 912.64087836553$   
A# -> G :  $16/9 \rightarrow 3/2 = 2^*3^*3^*3 / 2^*2^*2^*2^*2 = 54 / 32 = 905.86500259616$   
H -> G# :  $15/8 \rightarrow 30/19 = 2^*2^*3^*5^*2^*2^*2 / 19^*3^*5 = 32 / 19 = 902.4869838677$

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2 x 884.4 1 x 894.5 2 x 895.7 2 x 902.5 1 x 905.9 3 x 907.3 1 x 912.6

statistics for interval minor seventh:

C -> A# :  $1 \rightarrow 16/9 = 2^*2^*2^*2^*1 / 3^*3^*1 = 16 / 9 = 996.08999826923$   
C# -> H :  $20/19 \rightarrow 15/8 = 3^*5^*19 / 2^*2^*2^*2^*2^*5 = 57 / 32 = 999.46801699769$   
D -> C :  $19/17 \rightarrow 1 = 2^*1^*17 / 1^*19 = 34 / 19 = 1007.4423933681$   
D# -> C# :  $45/38 \rightarrow 20/19 = 2^*2^*2^*5^*2^*19 / 19^*3^*3^*5 = 16 / 9 = 996.08999826923$   
E -> D :  $5/4 \rightarrow 19/17 = 2^*19^*2^*2 / 17^*5 = 152 / 85 = 1006.2438927671$   
F -> D# :  $4/3 \rightarrow 45/38 = 2^*3^*3^*5^*3 / 2^*19^*2^*2 = 270 / 152 = 994.66570032869$   
F# -> E :  $24/17 \rightarrow 5/4 = 2^*5^*17 / 2^*2^*2^*2^*2^*3 = 170 / 96 = 989.31412249985$   
G -> F :  $3/2 \rightarrow 4/3 = 2^*2^*2^*2 / 3^*3 = 16 / 9 = 996.08999826923$   
G# -> F# :  $30/19 \rightarrow 24/17 = 2^*2^*2^*2^*3^*19 / 17^*2^*3^*5 = 152 / 85 = 1006.2438927671$



$A \rightarrow G: 5/3 \rightarrow 3/2 = 2^*3^*3 / 2^*5 = 18 / 10 = 1017.5962878659$   
 $A\# \rightarrow G\#: 16/9 \rightarrow 30/19 = 2^*2^*3^*5^*3^*3 / 19^*2^*2^*2^*2 = 270 / 152 = 994.66570032869$   
 $H \rightarrow A: 15/8 \rightarrow 5/3 = 2^*5^*2^*2^*2 / 3^*3^*5 = 16 / 9 = 996.08999826923$

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1 x 989.3 2 x 994.7 4 x 996.1 1 x 999.5 2 x 1006.2 1 x 1007.4 1 x 1017.6

statistics for interval major seventh:

$C \rightarrow H: 1 \rightarrow 15/8 = 3^*5^*1 / 2^*2^*2^*1 = 15 / 8 = 1088.2687147302$   
 $C\# \rightarrow C: 20/19 \rightarrow 1 = 2^*1^*19 / 1^*2^*2^*5 = 38 / 20 = 1111.1993022675$   
 $D \rightarrow C\#: 19/17 \rightarrow 20/19 = 2^*2^*2^*5^*17 / 19^*19 = 680 / 361 = 1096.2430911006$   
 $D\# \rightarrow D: 45/38 \rightarrow 19/17 = 2^*19^*2^*19 / 17^*3^*3^*5 = 1444 / 765 = 1099.8469071686$   
 $E \rightarrow D\#: 5/4 \rightarrow 45/38 = 2^*3^*3^*5^*2^*2 / 2^*19^*5 = 36 / 19 = 1106.3969855985$   
 $F \rightarrow E: 4/3 \rightarrow 5/4 = 2^*5^*3 / 2^*2^*2^*2 = 30 / 16 = 1088.2687147302$   
 $F\# \rightarrow F: 24/17 \rightarrow 4/3 = 2^*2^*2^*17 / 3^*2^*2^*2^*3 = 34 / 18 = 1101.0454077696$   
 $G \rightarrow F\#: 3/2 \rightarrow 24/17 = 2^*2^*2^*2^*3^*2 / 17^*3 = 32 / 17 = 1095.0445904996$   
 $G\# \rightarrow G: 30/19 \rightarrow 3/2 = 2^*3^*19 / 2^*2^*3^*5 = 38 / 20 = 1111.1993022675$   
 $A \rightarrow G\#: 5/3 \rightarrow 30/19 = 2^*2^*3^*5^*3 / 19^*5 = 36 / 19 = 1106.3969855985$   
 $A\# \rightarrow A: 16/9 \rightarrow 5/3 = 2^*5^*3^*3 / 3^*2^*2^*2^*2 = 30 / 16 = 1088.2687147302$   
 $H \rightarrow A\#: 15/8 \rightarrow 16/9 = 2^*2^*2^*2^*2^*2^*2 / 3^*3^*3^*5 = 256 / 135 = 1107.821283539$

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3 x 1088.3 1 x 1095 1 x 1096.2 1 x 1099.8 1 x 1101 2 x 1106.4 1 x 1107.8 2 x 1111.2

## Pure Tuning

C : 1 = 0  
C# : 19/18 = 93.603014401528  
D : 272/243 = 195.18040517347  
D# : 19/16 = 297.5130161323  
E : 361/288 = 391.11603053383  
F : 4/3 = 498.04499913461  
F# : 38/27 = 591.64801353614  
G : 256/171 = 698.57698213692  
G# : 19/12 = 795.55801526692  
A : 57/34 = 894.51260749728  
A# : 16/9 = 996.08999826923  
H : 361/192 = 1093.0710313992

statistics for interval minor second:

C -> C# : 1 -> 19/18 =  $19^*1 / 2^*3^*3^*1 = 19 / 18 = 93.603014401528$   
C# -> D : 19/18 -> 272/243 =  $2^*2^*2^*2^*17^*2^*3^*3 / 3^*3^*3^*3^*3^*19 = 544 / 513 = 101.57739077194$   
D -> D# : 272/243 -> 19/16 =  $19^*3^*3^*3^*3^*3 / 2^*2^*2^*2^*2^*2^*2^*17 = 4617 / 4352 = 102.33261095883$   
D# -> E : 19/16 -> 361/288 =  $19^*19^*2^*2^*2^*2 / 2^*2^*2^*2^*3^*3^*19 = 19 / 18 = 93.603014401528$   
E -> F : 361/288 -> 4/3 =  $2^*2^*2^*2^*2^*2^*3^*3 / 3^*19^*19 = 384 / 361 = 106.92896860078$   
F -> F# : 4/3 -> 38/27 =  $2^*19^*3 / 3^*3^*3^*2^*2 = 19 / 18 = 93.603014401528$   
F# -> G : 38/27 -> 256/171 =  $2^*2^*2^*2^*2^*2^*2^*3^*3^*3 / 3^*3^*19^*2^*19 = 384 / 361 = 106.92896860078$   
G -> G# : 256/171 -> 19/12 =  $19^*3^*3^*19 / 2^*2^*3^*2^*2^*2^*2^*2^*2^*2 = 1083 / 1024 = 96.981033129993$   
G# -> A : 19/12 -> 57/34 =  $3^*19^*2^*2^*3 / 2^*17^*19 = 18 / 17 = 98.954592230368$   
A -> A# : 57/34 -> 16/9 =  $2^*2^*2^*2^*2^*17 / 3^*3^*3^*19 = 544 / 513 = 101.57739077194$   
A# -> H : 16/9 -> 361/192 =  $19^*19^*3^*3 / 2^*2^*2^*2^*2^*3^*2^*2^*2^*2 = 1083 / 1024 = 96.981033129993$   
H -> C : 361/192 -> 1 =  $2^*1^*2^*2^*2^*2^*2^*2^*3 / 1^*19^*19 = 384 / 361 = 106.92896860078$

---

3 x 93.6 2 x 97 1 x 99 2 x 101.6 1 x 102.3 3 x 106.9

statistics for interval major second:

C -> D : 1 -> 272/243 =  $2^*2^*2^*2^*17^*1 / 3^*3^*3^*3^*3^*1 = 272 / 243 = 195.18040517347$   
C# -> D# : 19/18 -> 19/16 =  $19^*2^*3^*3 / 2^*2^*2^*2^*19 = 9 / 8 = 203.91000173077$   
D -> E : 272/243 -> 361/288 =  $19^*19^*3^*3^*3^*3^*3 / 2^*2^*2^*2^*3^*3^*2^*2^*2^*2^*17 = 9747 / 8704 = 195.93562536036$

D# -> F:  $19/16 \rightarrow 4/3 = 2^*2^*2^*2^*2^*2 / 3^*19 = 64 / 57 = 200.53198300231$   
 E -> F#:  $361/288 \rightarrow 38/27 = 2^*19^*2^*2^*2^*2^*2^*3^*3 / 3^*3^*3^*19^*19 = 64 / 57 = 200.53198300231$   
 F -> G:  $4/3 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*3 / 3^*3^*19^*2^*2 = 64 / 57 = 200.53198300231$   
 F# -> G#:  $38/27 \rightarrow 19/12 = 19^*3^*3^*3 / 2^*2^*3^*2^*19 = 9 / 8 = 203.91000173077$   
 G -> A:  $256/171 \rightarrow 57/34 = 3^*19^*3^*3^*19 / 2^*17^*2^*2^*2^*2^*2^*2^*2 = 9747 / 8704 = 195.93562536036$   
 G# -> A#:  $19/12 \rightarrow 16/9 = 2^*2^*2^*2^*2^*2^*3 / 3^*3^*19 = 64 / 57 = 200.53198300231$   
 A -> H:  $57/34 \rightarrow 361/192 = 19^*19^*2^*17 / 2^*2^*2^*2^*2^*2^*3^*3^*19 = 323 / 288 = 198.55842390193$   
 A# -> C:  $16/9 \rightarrow 1 = 2^*1^*3^*3 / 1^*2^*2^*2^*2 = 18 / 16 = 203.91000173077$   
 H -> C#:  $361/192 \rightarrow 19/18 = 2^*19^*2^*2^*2^*2^*2^*2^*3 / 2^*3^*3^*19^*19 = 64 / 57 = 200.53198300231$

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1 x 195.2 2 x 195.9 1 x 198.6 5 x 200.5 3 x 203.9

statistics for interval minor third:

C -> D#:  $1 \rightarrow 19/16 = 19^*1 / 2^*2^*2^*2^*1 = 19 / 16 = 297.5130161323$   
 C# -> E:  $19/18 \rightarrow 361/288 = 19^*19^*2^*3^*3 / 2^*2^*2^*2^*2^*3^*3^*19 = 19 / 16 = 297.5130161323$   
 D -> F:  $272/243 \rightarrow 4/3 = 2^*2^*3^*3^*3^*3^*3 / 3^*2^*2^*2^*2^*17 = 81 / 68 = 302.86459396114$   
 D# -> F#:  $19/16 \rightarrow 38/27 = 2^*19^*2^*2^*2^*2 / 3^*3^*3^*19 = 32 / 27 = 294.13499740384$   
 E -> G:  $361/288 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*2^*2^*2^*2^*3^*3 / 3^*3^*19^*19^*19 = 8192 / 6859 = 307.46095160309$   
 F -> G#:  $4/3 \rightarrow 19/12 = 19^*3 / 2^*2^*3^*2^*2 = 19 / 16 = 297.5130161323$   
 F# -> A:  $38/27 \rightarrow 57/34 = 3^*19^*3^*3^*3 / 2^*17^*2^*19 = 81 / 68 = 302.86459396114$   
 G -> A#:  $256/171 \rightarrow 16/9 = 2^*2^*2^*2^*3^*3^*19 / 3^*3^*2^*2^*2^*2^*2^*2^*2 = 19 / 16 = 297.5130161323$   
 G# -> H:  $19/12 \rightarrow 361/192 = 19^*19^*2^*2^*3 / 2^*2^*2^*2^*2^*2^*3^*19 = 19 / 16 = 297.5130161323$   
 A -> C:  $57/34 \rightarrow 1 = 2^*1^*2^*17 / 1^*3^*19 = 68 / 57 = 305.48739250272$   
 A# -> C#:  $16/9 \rightarrow 19/18 = 2^*19^*3^*3 / 2^*3^*3^*2^*2^*2^*2 = 38 / 32 = 297.5130161323$   
 H -> D:  $361/192 \rightarrow 272/243 = 2^*2^*2^*2^*2^*17^*2^*2^*2^*2^*2^*2^*3 / 3^*3^*3^*3^*3^*19^*19 = 34816 / 29241 = 302.10937377425$

---

1 x 294.1 6 x 297.5 1 x 302.1 2 x 302.9 1 x 305.5 1 x 307.5

statistics for interval major third:

C -> E:  $1 \rightarrow 361/288 = 19^*19^*1 / 2^*2^*2^*2^*2^*3^*3^*1 = 361 / 288 = 391.11603053383$   
 C# -> F:  $19/18 \rightarrow 4/3 = 2^*2^*2^*3^*3 / 3^*19 = 24 / 19 = 404.44198473308$   
 D -> F#:  $272/243 \rightarrow 38/27 = 2^*19^*3^*3^*3^*3^*3 / 3^*3^*3^*2^*2^*2^*2^*17 = 171 / 136 = 396.46760836267$   
 D# -> G:  $19/16 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*2^*2^*2^*2 / 3^*3^*19^*19 = 4096 / 3249 = 401.06396600462$

E -> G# :  $361/288 \rightarrow 19/12 = 19^*2^*2^*2^*2^*2^*3^*3 / 2^*2^*3^*19^*19 = 24 / 19 = 404.44198473308$   
 F -> A :  $4/3 \rightarrow 57/34 = 3^*19^*3 / 2^*17^*2^*2 = 171 / 136 = 396.46760836267$   
 F# -> A# :  $38/27 \rightarrow 16/9 = 2^*2^*2^*2^*3^*3^*3 / 3^*3^*2^*19 = 24 / 19 = 404.44198473308$   
 G -> H :  $256/171 \rightarrow 361/192 = 19^*19^*3^*3^*19 / 2^*2^*2^*2^*2^*3^*2^*2^*2^*2^*2^*2^*2 = 20577 / 16384 = 394.4940492623$   
 G# -> C :  $19/12 \rightarrow 1 = 2^*1^*2^*2^*3 / 1^*19 = 24 / 19 = 404.44198473308$   
 A -> C# :  $57/34 \rightarrow 19/18 = 2^*19^*2^*17 / 2^*3^*3^*3^*19 = 34 / 27 = 399.09040690424$   
 A# -> D :  $16/9 \rightarrow 272/243 = 2^*2^*2^*2^*2^*17^*3^*3 / 3^*3^*3^*3^*2^*2^*2^*2 = 34 / 27 = 399.09040690425$   
 H -> D# :  $361/192 \rightarrow 19/16 = 2^*19^*2^*2^*2^*2^*2^*3 / 2^*2^*2^*2^*19^*19 = 24 / 19 = 404.44198473308$

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1 x 391.1 1 x 394.5 2 x 396.5 2 x 399.1 1 x 401.1 5 x 404.4

statistics for interval fourth:

C -> F :  $1 \rightarrow 4/3 = 2^*2^*1 / 3^*1 = 4 / 3 = 498.04499913461$   
 C# -> F# :  $19/18 \rightarrow 38/27 = 2^*19^*2^*3^*3 / 3^*3^*3^*19 = 4 / 3 = 498.04499913461$   
 D -> G :  $272/243 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*3^*3^*3^*3 / 3^*3^*19^*2^*2^*2^*17 = 432 / 323 = 503.39657696345$   
 D# -> G# :  $19/16 \rightarrow 19/12 = 19^*2^*2^*2^*2 / 2^*2^*3^*19 = 4 / 3 = 498.04499913461$   
 E -> A :  $361/288 \rightarrow 57/34 = 3^*19^*2^*2^*2^*2^*2^*3^*3 / 2^*17^*19^*19 = 432 / 323 = 503.39657696345$   
 F -> A# :  $4/3 \rightarrow 16/9 = 2^*2^*2^*2^*3 / 3^*3^*2^*2 = 4 / 3 = 498.04499913461$   
 F# -> H :  $38/27 \rightarrow 361/192 = 19^*19^*3^*3^*3 / 2^*2^*2^*2^*2^*3^*2^*19 = 171 / 128 = 501.42301786308$   
 G -> C :  $256/171 \rightarrow 1 = 2^*1^*3^*3^*19 / 1^*2^*2^*2^*2^*2^*2^*2 = 342 / 256 = 501.42301786308$   
 G# -> C# :  $19/12 \rightarrow 19/18 = 2^*19^*2^*2^*3 / 2^*3^*3^*19 = 4 / 3 = 498.04499913461$   
 A -> D :  $57/34 \rightarrow 272/243 = 2^*2^*2^*2^*2^*17^*2^*17 / 3^*3^*3^*3^*3^*19 = 18496 / 13851 = 500.66779767619$   
 A# -> D# :  $16/9 \rightarrow 19/16 = 2^*19^*3^*3 / 2^*2^*2^*2^*2^*2^*2 = 342 / 256 = 501.42301786308$   
 H -> E :  $361/192 \rightarrow 361/288 = 2^*19^*19^*2^*2^*2^*2^*2^*3 / 2^*2^*2^*2^*3^*3^*19^*19 = 4 / 3 = 498.04499913461$

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6 x 498 1 x 500.7 3 x 501.4 2 x 503.4

statistics for interval tritone:

C -> F# :  $1 \rightarrow 38/27 = 2^*19^*1 / 3^*3^*3^*1 = 38 / 27 = 591.64801353614$   
 C# -> G :  $19/18 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*3^*3 / 3^*3^*19^*19 = 512 / 361 = 604.97396773539$   
 D -> G# :  $272/243 \rightarrow 19/12 = 19^*3^*3^*3^*3^*3 / 2^*2^*3^*2^*2^*2^*2^*17 = 1539 / 1088 = 600.37761009345$   
 D# -> A :  $19/16 \rightarrow 57/34 = 3^*19^*2^*2^*2^*2 / 2^*17^*19 = 24 / 17 = 596.99959136498$   
 E -> A# :  $361/288 \rightarrow 16/9 = 2^*2^*2^*2^*2^*2^*2^*3^*3 / 3^*3^*19^*19 = 512 / 361 =$

604.97396773539

$$F \rightarrow H: 4/3 \rightarrow 361/192 = 19^*19^*3 / 2^*2^*2^*2^*2^*3^*2^*2 = 361 / 256 =$$

595.02603226461

$$F\# \rightarrow C: 38/27 \rightarrow 1 = 2^*1^*3^*3^*3 / 1^*2^*19 = 54 / 38 = 608.35198646386$$

$$G \rightarrow C\#: 256/171 \rightarrow 19/18 = 2^*19^*3^*3^*19 / 2^*3^*3^*2^*2^*2^*2^*2^*2^*2 = 722 / 512 =$$

595.02603226461

$$G\# \rightarrow D: 19/12 \rightarrow 272/243 = 2^*2^*2^*2^*17^*2^*2^*3 / 3^*3^*3^*3^*19 = 2176 / 1539 =$$

599.62238990655

$$A \rightarrow D\#: 57/34 \rightarrow 19/16 = 2^*19^*2^*17 / 2^*2^*2^*2^*3^*19 = 34 / 24 = 603.00040863502$$

$$A\# \rightarrow E: 16/9 \rightarrow 361/288 = 2^*19^*19^*3^*3 / 2^*2^*2^*2^*2^*3^*3^*2^*2^*2^*2 = 722 / 512 =$$

595.02603226461

$$H \rightarrow F: 361/192 \rightarrow 4/3 = 2^*2^*2^*2^*2^*2^*2^*2^*3 / 3^*19^*19 = 512 / 361 =$$

604.97396773539

1 x 591.6 3 x 595 1 x 597 1 x 599.6 1 x 600.4 1 x 603 3 x 605 1 x 608.4

statistics for interval fifth:

$$C \rightarrow G: 1 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*1 / 3^*3^*19^*1 = 256 / 171 = 698.57698213692$$

$$C\# \rightarrow G\#: 19/18 \rightarrow 19/12 = 19^*2^*3^*3 / 2^*2^*3^*19 = 3 / 2 = 701.95500086539$$

$$D \rightarrow A: 272/243 \rightarrow 57/34 = 3^*19^*3^*3^*3^*3^*3 / 2^*17^*2^*2^*2^*2^*17 = 13851 / 9248 =$$

699.33220232381

$$D\# \rightarrow A\#: 19/16 \rightarrow 16/9 = 2^*2^*2^*2^*2^*2^*2 / 3^*3^*19 = 256 / 171 = 698.57698213692$$

$$E \rightarrow H: 361/288 \rightarrow 361/192 = 19^*19^*2^*2^*2^*2^*2^*3^*3 / 2^*2^*2^*2^*2^*3^*19^*19 = 3 / 2 =$$

701.95500086539

$$F \rightarrow C: 4/3 \rightarrow 1 = 2^*1^*3 / 1^*2^*2 = 6 / 4 = 701.95500086539$$

$$F\# \rightarrow C\#: 38/27 \rightarrow 19/18 = 2^*19^*3^*3^*3 / 2^*3^*3^*2^*19 = 6 / 4 = 701.95500086539$$

$$G \rightarrow D: 256/171 \rightarrow 272/243 = 2^*2^*2^*2^*2^*17^*3^*3^*19 / 3^*3^*3^*3^*2^*2^*2^*2^*2^*2^*2 =$$

646 / 432 = 696.60342303655

$$G\# \rightarrow D\#: 19/12 \rightarrow 19/16 = 2^*19^*2^*2^*3 / 2^*2^*2^*2^*19 = 6 / 4 = 701.95500086539$$

$$A \rightarrow E: 57/34 \rightarrow 361/288 = 2^*19^*19^*2^*17 / 2^*2^*2^*2^*2^*3^*3^*3^*19 = 646 / 432 =$$

696.60342303655

$$A\# \rightarrow F: 16/9 \rightarrow 4/3 = 2^*2^*2^*3^*3 / 3^*2^*2^*2^*2 = 6 / 4 = 701.95500086539$$

$$H \rightarrow F\#: 361/192 \rightarrow 38/27 = 2^*2^*19^*2^*2^*2^*2^*2^*3 / 3^*3^*3^*19^*19 = 256 / 171 =$$

698.57698213692

2 x 696.6 3 x 698.6 1 x 699.3 6 x 702

statistics for interval minor sixth:

$$C \rightarrow G\#: 1 \rightarrow 19/12 = 19^*1 / 2^*2^*3^*1 = 19 / 12 = 795.55801526692$$

$$C\# \rightarrow A: 19/18 \rightarrow 57/34 = 3^*19^*2^*3^*3 / 2^*17^*19 = 27 / 17 = 800.90959309576$$

$$D \rightarrow A\#: 272/243 \rightarrow 16/9 = 2^*2^*2^*2^*3^*3^*3^*3^*3 / 3^*3^*2^*2^*2^*2^*17 = 27 / 17 =$$

800.90959309575

$$D\# \rightarrow H: 19/16 \rightarrow 361/192 = 19^*19^*2^*2^*2^*2 / 2^*2^*2^*2^*2^*3^*19 = 19 / 12 =$$

795.55801526692

$$E \rightarrow C: 361/288 \rightarrow 1 = 2^1 \cdot 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 \cdot 3^3 / 1^1 \cdot 19^1 \cdot 19 = 576 / 361 = 808.88396946617$$

$$F \rightarrow C\#: 4/3 \rightarrow 19/18 = 2^1 \cdot 19^1 \cdot 3 / 2^1 \cdot 3^1 \cdot 3^1 \cdot 2^1 \cdot 2 = 38 / 24 = 795.55801526692$$

$$F\# \rightarrow D: 38/27 \rightarrow 272/243 = 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^1 \cdot 17^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 / 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 2^1 \cdot 19 = 272 / 171 = 803.53239163733$$

$$G \rightarrow D\#: 256/171 \rightarrow 19/16 = 2^1 \cdot 19^1 \cdot 3^1 \cdot 3^1 \cdot 19 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 = 6498 / 4096 = 798.93603399538$$

$$G\# \rightarrow E: 19/12 \rightarrow 361/288 = 2^1 \cdot 19^1 \cdot 19^1 \cdot 2^1 \cdot 2^1 \cdot 3 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 3^1 \cdot 19 = 38 / 24 = 795.55801526692$$

$$A \rightarrow F: 57/34 \rightarrow 4/3 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 17 / 3^1 \cdot 3^1 \cdot 19 = 272 / 171 = 803.53239163733$$

$$A\# \rightarrow F\#: 16/9 \rightarrow 38/27 = 2^1 \cdot 2^1 \cdot 19^1 \cdot 3^1 \cdot 3^1 / 3^1 \cdot 3^1 \cdot 3^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 = 38 / 24 = 795.55801526692$$

$$H \rightarrow G: 361/192 \rightarrow 256/171 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3 / 3^1 \cdot 3^1 \cdot 19^1 \cdot 19^1 \cdot 19 = 32768 / 20577 = 805.5059507377$$

5 x 795.6 1 x 798.9 2 x 800.9 2 x 803.5 1 x 805.5 1 x 808.9

statistics for interval major sixth:

$$C \rightarrow A: 1 \rightarrow 57/34 = 3^1 \cdot 19^1 \cdot 1 / 2^1 \cdot 17^1 \cdot 1 = 57 / 34 = 894.51260749728$$

$$C\# \rightarrow A\#: 19/18 \rightarrow 16/9 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 3^1 / 3^1 \cdot 3^1 \cdot 19 = 32 / 19 = 902.4869838677$$

$$D \rightarrow H: 272/243 \rightarrow 361/192 = 19^1 \cdot 19^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 17 = 29241 / 17408 = 897.89062622575$$

$$D\# \rightarrow C: 19/16 \rightarrow 1 = 2^1 \cdot 1^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 / 1^1 \cdot 19 = 32 / 19 = 902.4869838677$$

$$E \rightarrow C\#: 361/288 \rightarrow 19/18 = 2^1 \cdot 19^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 3^1 / 2^1 \cdot 3^1 \cdot 3^1 \cdot 19^1 \cdot 19 = 32 / 19 = 902.4869838677$$

$$F \rightarrow D: 4/3 \rightarrow 272/243 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 17^1 \cdot 3 / 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 2^1 \cdot 2^1 = 136 / 81 = 897.13540603886$$

$$F\# \rightarrow D\#: 38/27 \rightarrow 19/16 = 2^1 \cdot 19^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 19 = 54 / 32 = 905.86500259616$$

$$G \rightarrow E: 256/171 \rightarrow 361/288 = 2^1 \cdot 19^1 \cdot 19^1 \cdot 3^1 \cdot 3^1 \cdot 19 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 3^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 = 13718 / 8192 = 892.53904839691$$

$$G\# \rightarrow F: 19/12 \rightarrow 4/3 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3 / 3^1 \cdot 19 = 32 / 19 = 902.4869838677$$

$$A \rightarrow F\#: 57/34 \rightarrow 38/27 = 2^1 \cdot 2^1 \cdot 19^1 \cdot 2^1 \cdot 17 / 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 19 = 136 / 81 = 897.13540603886$$

$$A\# \rightarrow G: 16/9 \rightarrow 256/171 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 3^1 / 3^1 \cdot 3^1 \cdot 19^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 = 32 / 19 = 902.4869838677$$

$$H \rightarrow G\#: 361/192 \rightarrow 19/12 = 2^1 \cdot 19^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3 / 2^1 \cdot 2^1 \cdot 3^1 \cdot 19^1 \cdot 19 = 32 / 19 = 902.4869838677$$

1 x 892.5 1 x 894.5 2 x 897.1 1 x 897.9 6 x 902.5 1 x 905.9

statistics for interval minor seventh:

$$C \rightarrow A\#: 1 \rightarrow 16/9 = 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 1 / 3^1 \cdot 3^1 \cdot 1 = 16 / 9 = 996.08999826923$$

$$C\# \rightarrow H: 19/18 \rightarrow 361/192 = 19^1 \cdot 19^1 \cdot 2^1 \cdot 3^1 \cdot 3^1 / 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 3^1 \cdot 19 = 57 / 32 = 999.46801699769$$

$$D \rightarrow C: 272/243 \rightarrow 1 = 2^1 \cdot 1^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 \cdot 3^1 / 1^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 2^1 \cdot 17 = 486 / 272 =$$

1004.8195948265

$$D\# \rightarrow C\# : 19/16 \rightarrow 19/18 = 2^*19^*2^*2^*2^*2 / 2^*3^*3^*19 = 16 / 9 = 996.08999826923$$

$$E \rightarrow D : 361/288 \rightarrow 272/243 = 2^*2^*2^*2^*2^*17^*2^*2^*2^*2^*3^*3 / 3^*3^*3^*3^*3^*19^*19 = 17408 / 9747 = 1004.0643746396$$

$$F \rightarrow D\# : 4/3 \rightarrow 19/16 = 2^*19^*3 / 2^*2^*2^*2^*2^*2 = 114 / 64 = 999.46801699769$$

$$F\# \rightarrow E : 38/27 \rightarrow 361/288 = 2^*19^*19^*3^*3^*3 / 2^*2^*2^*2^*2^*3^*3^*2^*19 = 114 / 64 = 999.46801699769$$

$$G \rightarrow F : 256/171 \rightarrow 4/3 = 2^*2^*2^*3^*3^*19 / 3^*2^*2^*2^*2^*2^*2^*2^*2 = 114 / 64 = 999.46801699769$$

$$G\# \rightarrow F\# : 19/12 \rightarrow 38/27 = 2^*2^*19^*2^*2^*3 / 3^*3^*3^*19 = 16 / 9 = 996.08999826923$$

$$A \rightarrow G : 57/34 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*2^*2^*17 / 3^*3^*19^*3^*19 = 17408 / 9747 = 1004.0643746396$$

$$A\# \rightarrow G\# : 16/9 \rightarrow 19/12 = 2^*19^*3^*3 / 2^*2^*3^*2^*2^*2^*2 = 114 / 64 = 999.46801699769$$

$$H \rightarrow A : 361/192 \rightarrow 57/34 = 2^*3^*19^*2^*2^*2^*2^*2^*2^*3 / 2^*17^*19^*19 = 576 / 323 = 1001.4415760981$$

---

3 x 996.1 5 x 999.5 1 x 1001.4 2 x 1004.1 1 x 1004.8

statistics for interval major seventh:

$$C \rightarrow H : 1 \rightarrow 361/192 = 19^*19^*1 / 2^*2^*2^*2^*2^*2^*3^*1 = 361 / 192 = 1093.0710313992$$

$$C\# \rightarrow C : 19/18 \rightarrow 1 = 2^*1^*2^*3^*3 / 1^*19 = 36 / 19 = 1106.3969855985$$

$$D \rightarrow C\# : 272/243 \rightarrow 19/18 = 2^*19^*3^*3^*3^*3^*3 / 2^*3^*3^*2^*2^*2^*2^*17 = 1026 / 544 = 1098.4226092281$$

$$D\# \rightarrow D : 19/16 \rightarrow 272/243 = 2^*2^*2^*2^*2^*17^*2^*2^*2^*2 / 3^*3^*3^*3^*3^*19 = 8704 / 4617 = 1097.6673890412$$

$$E \rightarrow D\# : 361/288 \rightarrow 19/16 = 2^*19^*2^*2^*2^*2^*2^*3^*3 / 2^*2^*2^*2^*19^*19 = 36 / 19 = 1106.3969855985$$

$$F \rightarrow E : 4/3 \rightarrow 361/288 = 2^*19^*19^*3 / 2^*2^*2^*2^*2^*3^*3^*2^*2 = 722 / 384 = 1093.0710313992$$

$$F\# \rightarrow F : 38/27 \rightarrow 4/3 = 2^*2^*2^*3^*3^*3 / 3^*2^*19 = 36 / 19 = 1106.3969855985$$

$$G \rightarrow F\# : 256/171 \rightarrow 38/27 = 2^*2^*19^*3^*3^*19 / 3^*3^*3^*2^*2^*2^*2^*2^*2^*2 = 722 / 384 = 1093.0710313992$$

$$G\# \rightarrow G : 19/12 \rightarrow 256/171 = 2^*2^*2^*2^*2^*2^*2^*2^*2^*2^*3 / 3^*3^*19^*19 = 2048 / 1083 = 1103.01896687$$

$$A \rightarrow G\# : 57/34 \rightarrow 19/12 = 2^*19^*2^*17 / 2^*2^*3^*3^*19 = 34 / 18 = 1101.0454077696$$

$$A\# \rightarrow A : 16/9 \rightarrow 57/34 = 2^*3^*19^*3^*3 / 2^*17^*2^*2^*2^*2 = 1026 / 544 = 1098.4226092281$$

$$H \rightarrow A\# : 361/192 \rightarrow 16/9 = 2^*2^*2^*2^*2^*2^*2^*2^*2^*2^*3 / 3^*3^*19^*19 = 2048 / 1083 = 1103.01896687$$

---

3 x 1093.1 1 x 1097.7 2 x 1098.4 1 x 1101 2 x 1103 3 x 1106.4

## Werckmeister III temperament

C : 0  
C# : 90.225  
D : 192.18  
D# : 294.135  
E : 390.225  
F : 498.045  
F# : 588.27  
G : 696.09  
G# : 792.18  
A : 888.27  
A# : 996.09  
H : 1092.18

statistics for interval minor second:

C -> C# : 90.225  
C# -> D : 101.955  
D -> D# : 101.955  
D# -> E : 96.09  
E -> F : 107.82  
F -> F# : 90.225  
F# -> G : 107.82  
G -> G# : 96.09  
G# -> A : 96.09  
A -> A# : 107.82  
A# -> H : 96.09  
H -> C : 107.82

---

2 x 90.2 4 x 96.1 2 x 102 4 x 107.8

statistics for interval major second:

C -> D : 192.18  
C# -> D# : 203.91  
D -> E : 198.045  
D# -> F : 203.91  
E -> F# : 198.045  
F -> G : 198.045  
F# -> G# : 203.91  
G -> A : 192.18  
G# -> A# : 203.91  
A -> H : 203.91  
A# -> C : 203.91  
H -> C# : 198.045

---



2 x 192.2 4 x 198 6 x 203.9

statistics for interval minor third:

C -> D# : 294.135  
C# -> E : 300  
D -> F : 305.865  
D# -> F# : 294.135  
E -> G : 305.865  
F -> G# : 294.135  
F# -> A : 300  
G -> A# : 300  
G# -> H : 300  
A -> C : 311.73  
A# -> C# : 294.135  
H -> D : 300

---

4 x 294.1 5 x 300 2 x 305.9 1 x 311.7

statistics for interval major third:

C -> E : 390.225  
C# -> F : 407.82  
D -> F# : 396.09  
D# -> G : 401.955  
E -> G# : 401.955  
F -> A : 390.225  
F# -> A# : 407.82  
G -> H : 396.09  
G# -> C : 407.82  
A -> C# : 401.955  
A# -> D : 396.09  
H -> D# : 401.955

---

2 x 390.2 3 x 396.1 4 x 402 3 x 407.8

statistics for interval fourth:

C -> F : 498.045  
C# -> F# : 498.045  
D -> G : 503.91  
D# -> G# : 498.045  
E -> A : 498.045  
F -> A# : 498.045  
F# -> H : 503.91  
G -> C : 503.91

G# -> C# : 498.045  
A -> D : 503.91  
A# -> D# : 498.045  
H -> E : 498.045

---

8 x 498 4 x 503.9

statistics for interval tritone:

C -> F# : 588.27  
C# -> G : 605.865  
D -> G# : 600  
D# -> A : 594.135  
E -> A# : 605.865  
F -> H : 594.135  
F# -> C : 611.73  
G -> C# : 594.135  
G# -> D : 600  
A -> D# : 605.865  
A# -> E : 594.135  
H -> F : 605.865

---

1 x 588.3 4 x 594.1 2 x 600 4 x 605.9 1 x 611.7

statistics for interval fifth:

C -> G : 696.09  
C# -> G# : 701.955  
D -> A : 696.09  
D# -> A# : 701.955  
E -> H : 701.955  
F -> C : 701.955  
F# -> C# : 701.955  
G -> D : 696.09  
G# -> D# : 701.955  
A -> E : 701.955  
A# -> F : 701.955  
H -> F# : 696.09

---

4 x 696.1 8 x 702

statistics for interval minor sixth:

C -> G# : 792.18  
C# -> A : 798.045

D -> A# : 803.91  
D# -> H : 798.045  
E -> C : 809.775  
F -> C# : 792.18  
F# -> D : 803.91  
G -> D# : 798.045  
G# -> E : 798.045  
A -> F : 809.775  
A# -> F# : 792.18  
H -> G : 803.91

---

3 x 792.2 4 x 798 3 x 803.9 2 x 809.8

statistics for interval major sixth:

C -> A : 888.27  
C# -> A# : 905.865  
D -> H : 900  
D# -> C : 905.865  
E -> C# : 900  
F -> D : 894.135  
F# -> D# : 905.865  
G -> E : 894.135  
G# -> F : 905.865  
A -> F# : 900  
A# -> G : 900  
H -> G# : 900

---

1 x 888.3 2 x 894.1 5 x 900 4 x 905.9

statistics for interval minor seventh:

C -> A# : 996.09  
C# -> H : 1001.955  
D -> C : 1007.82  
D# -> C# : 996.09  
E -> D : 1001.955  
F -> D# : 996.09  
F# -> E : 1001.955  
G -> F : 1001.955  
G# -> F# : 996.09  
A -> G : 1007.82  
A# -> G# : 996.09  
H -> A : 996.09

---

6 x 996.1 4 x 1002 2 x 1007.8

statistics for interval major seventh:

C -> H : 1092.18  
C# -> C : 1109.775  
D -> C# : 1098.045  
D# -> D : 1098.045  
E -> D# : 1103.91  
F -> E : 1092.18  
F# -> F : 1109.775  
G -> F# : 1092.18  
G# -> G : 1103.91  
A -> G# : 1103.91  
A# -> A : 1092.18  
H -> A# : 1103.91

---

4 x 1092.2   2 x 1098   4 x 1103.9   2 x 1109.8

## Lehman-Bach temperament

C : 0  
C# : 98.045  
D : 196.09  
D# : 298.045  
E : 392.18  
F : 501.955  
F# : 596.09  
G : 698.045  
G# : 798.045  
A : 894.135  
A# : 998.045  
H : 1094.135

statistics for interval minor second:

C -> C# : 98.045  
C# -> D : 98.045  
D -> D# : 101.955  
D# -> E : 94.135  
E -> F : 109.775  
F -> F# : 94.135  
F# -> G : 101.955  
G -> G# : 100  
G# -> A : 96.09  
A -> A# : 103.91  
A# -> H : 96.09  
H -> C : 105.865

---

2 x 94.1 2 x 96.1 2 x 98 1 x 100 2 x 102 1 x 103.9 1 x 105.9 1 x 109.8

statistics for interval major second:

C -> D : 196.09  
C# -> D# : 200  
D -> E : 196.09  
D# -> F : 203.91  
E -> F# : 203.91  
F -> G : 196.09  
F# -> G# : 201.955  
G -> A : 196.09  
G# -> A# : 200  
A -> H : 200  
A# -> C : 201.955

H -> C# : 203.91

---

4 x 196.1 3 x 200 2 x 202 3 x 203.9

statistics for interval minor third:

C -> D# : 298.045  
C# -> E : 294.135  
D -> F : 305.865  
D# -> F# : 298.045  
E -> G : 305.865  
F -> G# : 296.09  
F# -> A : 298.045  
G -> A# : 300  
G# -> H : 296.09  
A -> C : 305.865  
A# -> C# : 300  
H -> D : 301.955

---

1 x 294.1 2 x 296.1 3 x 298 2 x 300 1 x 302 3 x 305.9

statistics for interval major third:

C -> E : 392.18  
C# -> F : 403.91  
D -> F# : 400  
D# -> G : 400  
E -> G# : 405.865  
F -> A : 392.18  
F# -> A# : 401.955  
G -> H : 396.09  
G# -> C : 401.955  
A -> C# : 403.91  
A# -> D : 398.045  
H -> D# : 403.91

---

2 x 392.2 1 x 396.1 1 x 398 2 x 400 2 x 402 3 x 403.9 1 x 405.9

statistics for interval fourth:

C -> F : 501.955  
C# -> F# : 498.045  
D -> G : 501.955  
D# -> G# : 500  
E -> A : 501.955

F -> A# : 496.09  
F# -> H : 498.045  
G -> C : 501.955  
G# -> C# : 500  
A -> D : 501.955  
A# -> D# : 500  
H -> E : 498.045

---

1 x 496.1 3 x 498 3 x 500 5 x 502

statistics for interval tritone:

C -> F# : 596.09  
C# -> G : 600  
D -> G# : 601.955  
D# -> A : 596.09  
E -> A# : 605.865  
F -> H : 592.18  
F# -> C : 603.91  
G -> C# : 600  
G# -> D : 598.045  
A -> D# : 603.91  
A# -> E : 594.135  
H -> F : 607.82

---

1 x 592.2 1 x 594.1 2 x 596.1 1 x 598 2 x 600 1 x 602 2 x 603.9 1 x 605.9 1 x 607.8

statistics for interval fifth:

C -> G : 698.045  
C# -> G# : 700  
D -> A : 698.045  
D# -> A# : 700  
E -> H : 701.955  
F -> C : 698.045  
F# -> C# : 701.955  
G -> D : 698.045  
G# -> D# : 700  
A -> E : 698.045  
A# -> F : 703.91  
H -> F# : 701.955

---

5 x 698 3 x 700 3 x 702 1 x 703.9

statistics for interval minor sixth:

C -> G# : 798.045  
C# -> A : 796.09  
D -> A# : 801.955  
D# -> H : 796.09  
E -> C : 807.82  
F -> C# : 796.09  
F# -> D : 800  
G -> D# : 800  
G# -> E : 794.135  
A -> F : 807.82  
A# -> F# : 798.045  
H -> G : 803.91

---

1 x 794.1 3 x 796.1 2 x 798 2 x 800 1 x 802 1 x 803.9 2 x 807.8

statistics for interval major sixth:

C -> A : 894.135  
C# -> A# : 900  
D -> H : 898.045  
D# -> C : 901.955  
E -> C# : 905.865  
F -> D : 894.135  
F# -> D# : 901.955  
G -> E : 894.135  
G# -> F : 903.91  
A -> F# : 901.955  
A# -> G : 900  
H -> G# : 903.91

---

3 x 894.1 1 x 898 2 x 900 3 x 902 2 x 903.9 1 x 905.9

statistics for interval minor seventh:

C -> A# : 998.045  
C# -> H : 996.09  
D -> C : 1003.91  
D# -> C# : 1000  
E -> D : 1003.91  
F -> D# : 996.09  
F# -> E : 996.09  
G -> F : 1003.91  
G# -> F# : 998.045  
A -> G : 1003.91  
A# -> G# : 1000  
H -> A : 1000



---

3 x 996.1 2 x 998 3 x 1000 4 x 1003.9

statistics for interval major seventh:

C -> H : 1094.135  
C# -> C : 1101.955  
D -> C# : 1101.955  
D# -> D : 1098.045  
E -> D# : 1105.865  
F -> E : 1090.225  
F# -> F : 1105.865  
G -> F# : 1098.045  
G# -> G : 1100  
A -> G# : 1103.91  
A# -> A : 1096.09  
H -> A# : 1103.91

---

1 x 1090.2 1 x 1094.1 1 x 1096.1 2 x 1098 1 x 1100 2 x 1102 2 x 1103.9 2 x 1105.9

## Equal temperament

C : 0  
C# : 100  
D : 200  
D# : 300  
E : 400  
F : 500  
F# : 600  
G : 700  
G# : 800  
A : 900  
A# : 1000  
H : 1100

statistics for interval minor second:

C -> C# : 100  
C# -> D : 100  
D -> D# : 100  
D# -> E : 100  
E -> F : 100  
F -> F# : 100  
F# -> G : 100  
G -> G# : 100  
G# -> A : 100  
A -> A# : 100  
A# -> H : 100  
H -> C : 100

---

12 x 100

statistics for interval major second:

C -> D : 200  
C# -> D# : 200  
D -> E : 200  
D# -> F : 200  
E -> F# : 200  
F -> G : 200  
F# -> G# : 200  
G -> A : 200  
G# -> A# : 200  
A -> H : 200  
A# -> C : 200  
H -> C# : 200

---

12 x 200

statistics for interval minor third:

C -> D# : 300  
C# -> E : 300  
D -> F : 300  
D# -> F# : 300  
E -> G : 300  
F -> G# : 300  
F# -> A : 300  
G -> A# : 300  
G# -> H : 300  
A -> C : 300  
A# -> C# : 300  
H -> D : 300

---

12 x 300

statistics for interval major third:

C -> E : 400  
C# -> F : 400  
D -> F# : 400  
D# -> G : 400  
E -> G# : 400  
F -> A : 400  
F# -> A# : 400  
G -> H : 400  
G# -> C : 400  
A -> C# : 400  
A# -> D : 400  
H -> D# : 400

---

12 x 400

statistics for interval fourth:

C -> F : 500  
C# -> F# : 500  
D -> G : 500  
D# -> G# : 500  
E -> A : 500  
F -> A# : 500  
F# -> H : 500  
G -> C : 500

G# -> C# : 500  
A -> D : 500  
A# -> D# : 500  
H -> E : 500

---

12 x 500

statistics for interval tritone:

C -> F# : 600  
C# -> G : 600  
D -> G# : 600  
D# -> A : 600  
E -> A# : 600  
F -> H : 600  
F# -> C : 600  
G -> C# : 600  
G# -> D : 600  
A -> D# : 600  
A# -> E : 600  
H -> F : 600

---

12 x 600

statistics for interval fifth:

C -> G : 700  
C# -> G# : 700  
D -> A : 700  
D# -> A# : 700  
E -> H : 700  
F -> C : 700  
F# -> C# : 700  
G -> D : 700  
G# -> D# : 700  
A -> E : 700  
A# -> F : 700  
H -> F# : 700

---

12 x 700

statistics for interval minor sixth:

C -> G# : 800  
C# -> A : 800

D -> A# : 800  
D# -> H : 800  
E -> C : 800  
F -> C# : 800  
F# -> D : 800  
G -> D# : 800  
G# -> E : 800  
A -> F : 800  
A# -> F# : 800  
H -> G : 800

---

12 x 800

statistics for interval major sixth:

C -> A : 900  
C# -> A# : 900  
D -> H : 900  
D# -> C : 900  
E -> C# : 900  
F -> D : 900  
F# -> D# : 900  
G -> E : 900  
G# -> F : 900  
A -> F# : 900  
A# -> G : 900  
H -> G# : 900

---

12 x 900

statistics for interval minor seventh:

C -> A# : 1000  
C# -> H : 1000  
D -> C : 1000  
D# -> C# : 1000  
E -> D : 1000  
F -> D# : 1000  
F# -> E : 1000  
G -> F : 1000  
G# -> F# : 1000  
A -> G : 1000  
A# -> G# : 1000  
H -> A : 1000

---

12 x 1000

statistics for interval major seventh:

C -> H : 1100  
C# -> C : 1100  
D -> C# : 1100  
D# -> D : 1100  
E -> D# : 1100  
F -> E : 1100  
F# -> F : 1100  
G -> F# : 1100  
G# -> G : 1100  
A -> G# : 1100  
A# -> A : 1100  
H -> A# : 1100

---

12 x 1100